B.Sc Part I (Physics Hons)
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Q Obtain ~~Gallieau~~ Galilean Transformation equation. How that the Newtonian laws of mechanics are the same in all inertial frame?

Ans (a) The consequences of research work of Galileo on the motion of the projectile led him to formulate transformations which later on were called after his name "Galilean Transformation". These are used to describe the motions which are observed by two observers in two different inertial frame. It's two main results are follows:—

(I) The motion of a particle projected at any angle may be derived from the motion of the particle thrown vertically upward.

(II) If the particle is thrown straight up from a cart which is moving with uniform speed, the observer on the cart may see the particle moving up and down but the motion observed by an observer on the ground may be described by super imposing the motion of the cart into that of projectile.

Consider two frame S and S' of references one at rest and the other is moving with uniform velocity V. Let O and O' be the observers situated at the origins of S and S' respectively. They are observing the same event at any point P. Let the two frames be parallel to each other i.e X' axis is parallel to x-axis, y'-axis is parallel to y-axis and z' axis is parallel to z-axis.

Let the co-ordinate of P be (x, y, z, t)

and (x', y', z', t') relative to origin O and O' respectively. The choice of the origins of two frames is such that their origins coincides at time $t=0, t'=0$

Case I :-

Let the fram S' have the velocity v only in x' direction. Then O' has velocity v only along x'-axis. The two systems can be combined to each other by following equation.

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \quad \text{------} \quad (1)$$

Case II :-

Let the frame S' have velocity along straight line in any direction such that

$$V = iV_x + jV_y + kV_z$$

After time t, the frame S' seperated from S by distances iV_x, iV_y, iV_z along x, y, z axis respectively. Then the two systems can be related by the following equations

$$\begin{aligned} x' &= x - iV_x \\ y' &= y - iV_y \\ z' &= z - iV_z \\ t' &= t \end{aligned} \quad \text{------} \quad (II)$$

Transformation (1) and (2) are called Galilean Transformations

Newtonian fundamental equations are in varient under Galilean transformations. We prove the assertion by taking Newton's second law of motion in the absolute system of co-ordeniates, a particle acted upon by a force F has an accl^m $\frac{d^2x}{dt^2}$ such that

$$F = m\frac{d^2x}{dt^2} \quad \text{------} \quad (III)$$

In Galilean frame of reference

$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$

Thus $$\frac{dx'}{dt} = \frac{dx}{dt} - V \quad dt' = dt$$

$$\Rightarrow \frac{dx'}{dt'} = \frac{dx}{dt} - V$$

$$\Rightarrow \frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2} \quad \text{as } V \text{ remain constant} \quad \text{---} \quad (IV)$$

In Newtonian mechanics forces and masses are absolute quantities so that

$$m' = m, \quad F' = F \quad \text{------} \quad (V)$$

Putting the value from (IV) and (V) in (3)

$$F = m\frac{d^2x'}{dt'^2} \quad \text{------} \quad (VI)$$

Comparing eq^n (III) & (VI) we can say that ~~Newtonian~~ Newton's second law of motion is invariant under Galilean Transformation.